Panel Data

MMRM = mixed models for repeated measures – biostatisticians – use PROC MIXED! Impose serial correlation/autocorrelation in errors for each individual (no dynamics in the model! – maybe a trend)

dummy variable for individual (so all observations for individual )

observed but constant over time (only varies across individuals)

observed and vary over both time and individuals

Bayesian version is BHM (Bayesian Hierarchical Model) or multi-level model, MLM

Panel model or longitudinal model

Fixed and random effects models

For statisticians, the X\_i are the fixed effects (fixed/constant over time) and the Zs are the ‘random effects” (vary over time)

For econometricians, the fixed effects model just includes the to capture unobserved heterogeneity across individuals (no specified relationship in possible coefficient estimates).

A random effects model specifies a distribution on the coefficients to capture the idea that individual unobserved effects are related in some way (come from the same distribution).

From a Bayesian viewpoint, a fixed effects model would be with a uniform (or uninformative) prior on the coefficients, whereas a random effects model specifies a more informative prior to capture the knowledge that individuals are similar in some respects, e.g., specify a Normal hierarchical prior for the coefficients: BHM or multi-level model

prior for

hierarchical prior for the mean of the s

Frequentist approach is to specify that the s come from some distribution with mean zero, so it can be treated as decomposing the error term, the error term becomes:

Frequentist ‘differencing’ of the panel model

If we difference our model (equation 1 above):

So all the ‘fixed effects’ are removed, but we induce serial correlation in the error and all the observed fixed effects (constant over time) are also removed.

[Instead, we could difference the variables first, as in time series models, and then build a model for these! Then we would include the variables that do not vary over time and dummy variables for individuals.]

Difference between a standard Bayesian model and a Hierarchical Bayesian model:

In a standard model, we would specify priors for all the coefficients with known values given for all the prior parameters, e.g.,

We need priors for all the parameters:

(supposing we have some prior info.

about these coefficients

So we put values in for all the prior parameters.

Hierarchical priors for and :

Replace the above priors with

hierarchical prior for the mean of the s with a hierarchical prior for the variance!

prior for

To estimate in practice, we usually stack (vectorize) the data: (suppose only 3 individuals and 3 time periods)

**The main issue** with the above specification is that it is static (the dynamics are not explicitly specified and addressed in the model).

Everyone just assumes all variables are stationary and tries to capture the dynamics by specifying serially correlated errors (a bad idea!).

We really want a dynamic panel model.

If there are only a few time series observations, then we lost a lot of data by differencing and including lags.

IF ANY of the variables are nonstationary, then we have the spurious regression problem!

Another issue then, is what does nonstationary/stationary mean in a panel?

Do we do unit root test for each individual? Could the panel be stationary even if individuals are not and vice versa?

Hard to know what is best to do? DEPENDS ON THE CONTEXT – we must **THINK** about the DGP we are trying to model.

The key is to avoid the spurious regression problem.

The good news is that we can test for stationarity in the panel in a different way than is possible with a time series.

We can estimate the mean and variance of the sample at each time point and evaluate whether or not the mean and/or variance are constant over time.

Comparison of means, comparison of variances (just use MC or MCMC).

Do unit root tests on individuals as well?

Example results using Bayes\_MMRM\_example.jl

n = 30

t = 4

rho = 0.7

si = 0.2

st = 0.2

b = [1.0; 1.0]

y2, x2 = mixed\_dgp2(si, st, rho, t, n, b)

y = reshape(y2, (t, n))

plot(y, label=false)

M = 2000

model\_m = bhm\_mixed(y2, x2, t)

@time cc = sample(model\_m, NUTS(0.65), M)

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│ Parameter │ Mean │ Std │ P-Val │ CI-95% │ HPD-95% │

│ s\_i │ 0.198 │ 0.085 │ 0.0 │ [0.023, 0.353] │ [0.014, 0.341] │

│ s\_t │ 0.29 │ 0.11 │ 0.0 │ [0.106, 0.529] │ [0.105, 0.523] │

│ rho │ 0.689 │ 0.168 │ 0.0 │ [0.344, 0.968] │ [0.392, 0.992] │

│ b[1] │ 0.934 │ 0.095 │ 0.0 │ [0.751, 1.119] │ [0.763, 1.127] │

│ b[2] │ 1.086 │ 0.093 │ 0.0 │ [0.904, 1.261] │ [0.891, 1.246] │

Simpler covariance structure:

model\_m = bhm\_simple(y2, x2, t)

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│ Parameter │ Mean │ Std │ P-Val │ CI-95% │ HPD-95% │

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│ s\_t │ 0.465 │ 0.062 │ 0.0 │ [0.36, 0.601] │ [0.343, 0.581] │

│ rho │ 0.494 │ 0.285 │ 0.0 │ [0.023, 0.969] │ [0.006, 0.946] │

│ b[1] │ 0.943 │ 0.066 │ 0.0 │ [0.818, 1.073] │ [0.812, 1.066] │

│ b[2] │ 1.085 │ 0.064 │ 0.0 │ [0.963, 1.211] │ [0.968, 1.216] │

Bayesian Hierarchical Model

**BHM set-up**

**Priors**

, is a numerical value

, is EITHER hierarchical priors, say or , OR

nonhierarchical) a numerical value.

(uniform with range from 0 to 50)

**Model/likelihood**

(n individuals)

(T time periods)

end

end

[Alternatively, we could vectorize over time to do something like:

for i in 1 to n

y[(i-1)\*t+1:i\*t] ~ MvNormal(xb[(i-1)\*t+1:i\*t], omega)

end

i.e., just stacking the T time observations for each individual ]